Control Systems Laboratory (EE 3321) — Experiment 9
Coupled Tank Hardware, State-coupled SISO system Using MATLAB

I. Overview of Experimental Procedure

For this experiment, the student will make a Simulink model using QUARC blocks to control the water level in tank 2. Using the principles learned in the below sections, the student will then design a model for level controller for tank 2. Additionally, the student will analyze the response of the system to determine the gains for the equipment used. This will allow the student to create a detailed transfer function which models the system, and the student will be able to compare the theoretical and actual responses in real time.

II. Tank 2-Nonlinear Equation of Motion (EOM)

This section explains the mathematical model of your Coupled-Tank system Tank 2, the pump feeds into tank 1, which in turn feeds into tank 2. As far as tank 1 is concerned, the same equations as the ones explained in previous Simulink lab will apply. However, the water level Equation Of Motion (EOM) in tank 2 still needs to be derived. The input to the tank 2 process is the water level, $L1$, in tank 1 (generating the outflow feeding tank 2) and its output variable is the water level, $L2$, in tank 2 (i.e. bottom tank). The purpose of the present modelling session is to guide you with the system's open-loop transfer function, $G2(s)$, which in turn will be used to design an appropriate level controller. The obtained EOM should be a function of the system's input and output, as previously defined.

Therefore, you should express the resulting EOM under the following format:

$$\frac{\partial L_2}{\partial t} = f(L_2, L_1) \quad 9.1$$

where f denotes a function.

In deriving the tank #2 EOM the mass balance principle can be applied to the water level in tank 2 as follows

$$A_{t2}\frac{\partial L_2}{\partial t} = F_{i2} - F_{o2} \quad 9.2$$

Where A_{t2} is the area of tank 2. F_{i2} and F_{o2} are the inflow rate and outflow rate, respectively.

The volumetric inflow rate to tank 2 is equal to the volumetric outflow rate from tank 1, that is to say:

$$F_{i2} = F_{o1} \quad 9.3$$

Applying Bernoulli's equation for small orifices, the outflow velocity from tank 2, $vo2$, can be expressed by the following relationship:

$$v_{o2} = \sqrt{2gL_2} \quad 9.4$$

III. Tank 2-EOM Linearization and Transfer Function

In order to design and implement a linear level controller for the tank 2 system, the Laplace open-loop transfer function should be derived. However by definition, such a transfer function can only represent the system's dynamics from a linear differential equation. Therefore, the nonlinear EOM of tank 2 should be linearized around a quiescent point of operation.

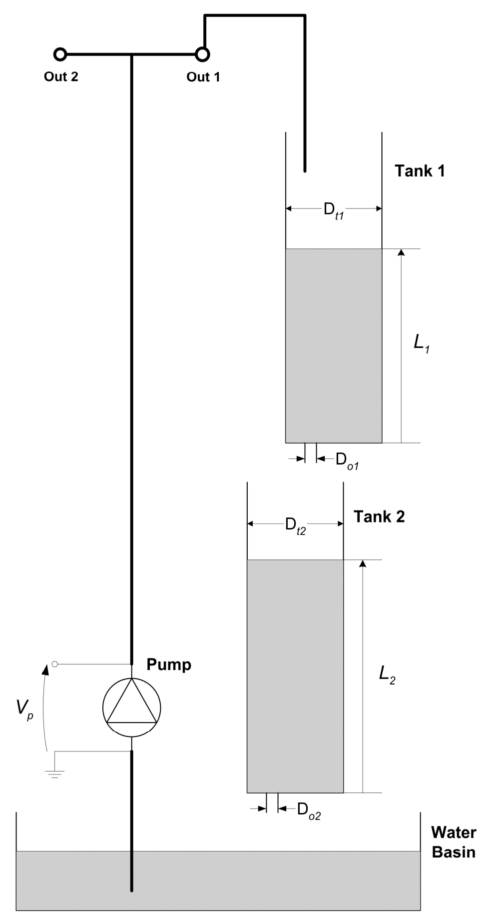
In the case of the water level in tank 2, the operating range corresponds to small departure heights, $L11$ and $L21$, from the desired equilibrium point ($L10;L20$). Therefore, $L2$ and $L1$ can be expressed as the sum of two quantities, as shown below:

$$L_2 = L_{20} + L_{21}, \qquad L_1 = L_{10} + L_{11}$$

The obtained linearized EOM should be a function of the system's small deviations about its equilibrium point (L20;L10). Therefore, you should express the resulting linear EOM under the following format:

$$\frac{\partial}{\partial t} L_{21} = f(L_{11}, L_{21}) \qquad 9.5$$

where f denotes a function.



For a function, f, of two variables, L1 and L2, a first-order approximation for small variations at a point (L1;L2) =(L10;L20) is given by the following Taylor's series approximation:

$$\frac{\partial^2}{\partial L_1 \partial L_2} f(L_1, L_2) \cong f(L_{10}, L_{20}) + \left(\frac{\partial}{\partial L_1} f(L_{10}, L_{20})\right) + (L_1 - L_{10}) \left(\frac{\partial}{\partial L_2} f(L_{10}, L_{20})\right) (L_2 - L_{20})$$

9.6

Transfer Function:

From the linear equation of motion, the system's open-loop transfer function in the Laplace domain can be defined by the following relationship:

$$G_2(s) = \frac{L_{21}(s)}{L_{11}(s)}$$

the desired open-loop transfer function for the Coupled-Tank's tank 2 system, such that:

$$G_2(s) = \frac{K_{dc2}}{\tau_2 s + 1} \qquad 9.7$$

where Kdc2 is the open-loop transfer function DC gain, and 2 is the time constant.

As a remark, it is obvious that linearized models, such as the Coupled-Tank's tank 2 level-to-level transfer function, are only approximate models. Therefore, they should be treated as such and used with appropriate caution that is to say within the valid operating range and/or conditions. However for the scope of this lab, Equation 2.10 is assumed valid over tank 1 and tank 2 water level entire range of motion, L1_max and L2_max, respectively.

IV. Tank 2 Level Control Design

For zero steady-state error, tank 1 water level is controlled by means of a Proportional-plus-Integral (PI) closed-loop scheme with the addition of a feedforward action, as illustrated in Figure 9.1, below.

In the block diagram depicted in Figure 9.1, the water level in tank 1 is controlled by means of the closed-loop system designed in previous experiment. This is represented by the tank 1 closed-loop transfer function defined below:

$$T_1(s) = \frac{L_1(s)}{L_{r\_1}(s)} \qquad 9.8$$

Such a subsystem represent t_{s_2} as an inner (or nested) level loop. In order to achieve a good overall stability with such a configuration, the inner level loop (i.e. tank 1 closed-loop system) must be much faster than the outer level loop. This constraint is met by the previously stated controller design specifications, where $t_{s\_1} < t_{s\_2}$.

However for the sake of simplicity in the present analysis, the water level dynamics in tank 1 are neglected. Therefore, it is assumed hereafter that:

$$L_1(t) = L_{r\_1}(t) \qquad i.e. \qquad T_1(s) = 1$$

Furthermore as depicted in Figure 9.1, the level feedforward action is characterized by:

$$L_{ff\_1} = K_{ff\_2} L_{r\_2} \qquad 9.9$$

and

$$L_1 = L_{11} + L_{ff\_1} \qquad 9.10$$

The level feedforward action, as seen in Figure 9.1, is necessary since the PI control system is only designed to compensate for small variations (a.k.a. disturbances) from the linearized operating point L10, L20. In other words, while the feedforward action compensates for the water withdrawal (due to gravity) through tank 2's bottom outlet orifice, the PI controller compensates for dynamic disturbances.

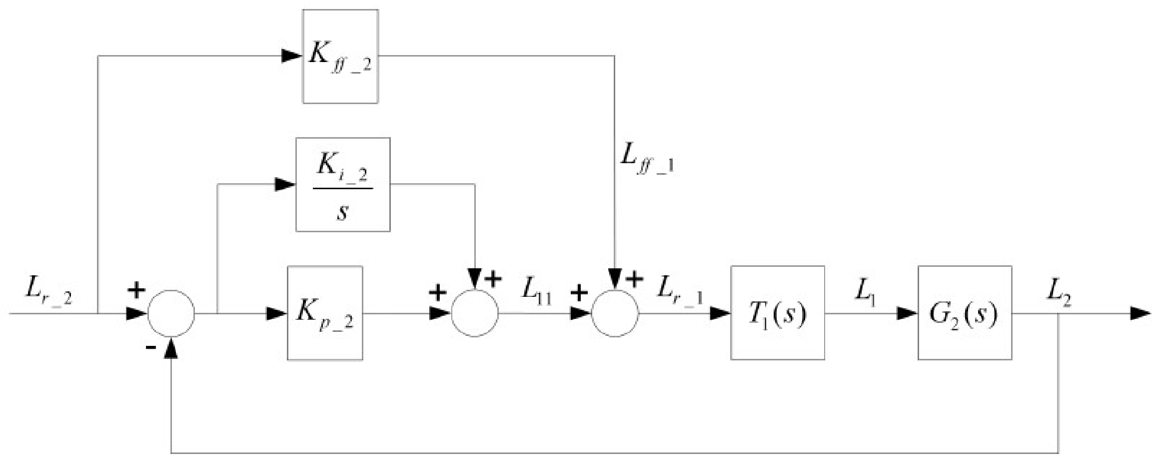


Figure 9.1: Tank 2 Water Level PI-plus-Feedforward Control Loop.

The open-loop transfer function G2(s) takes into account the dynamics of the tank 2 water level loop, as characterized by Equation 2.10. However, due to the presence of the feedforward loop and the simplifying assumption expressed by G2(s) can also be written as follows:

$$G_2(s) = \frac{L_2(s)}{L_1(s)} \quad 9.11$$

V. Tank 2 Level Control Simulation

In Tank2 level control, the pump feeds tank 1 and tank 1 feeds tank 2. The designed closed-loop system is to control the water level in tank 2 (i.e. the bottom tank) from the water flow coming out of tank 1, located above it. Similarly to Tank1, the control scheme is based on a Proportional-plus-Integral-plus-Feedforward law. In response to a desired ± 1 cm square wave level set point from tank 2 equilibrium level position, the water height behavior should satisfy the following design performance requirements:

1. Tank 2 operating level at 15 cm: *L*20 = 15 cm.
2. Percent overshoot should be less than or equal to 10%: *PO*2 _ 10*:*0 %.
3. 2% settling time less than 20 seconds: *ts,*2_20*:*0 s.
4. No steady-state error: *ess,2*= 0 cm.

The following steps should be followed for level control simulation:

The s_tanks_2 Simulink diagram shown in Figure 9.2 will be used to simulate the tank 2 level control response with the PI+FF controller used in earlier Section.

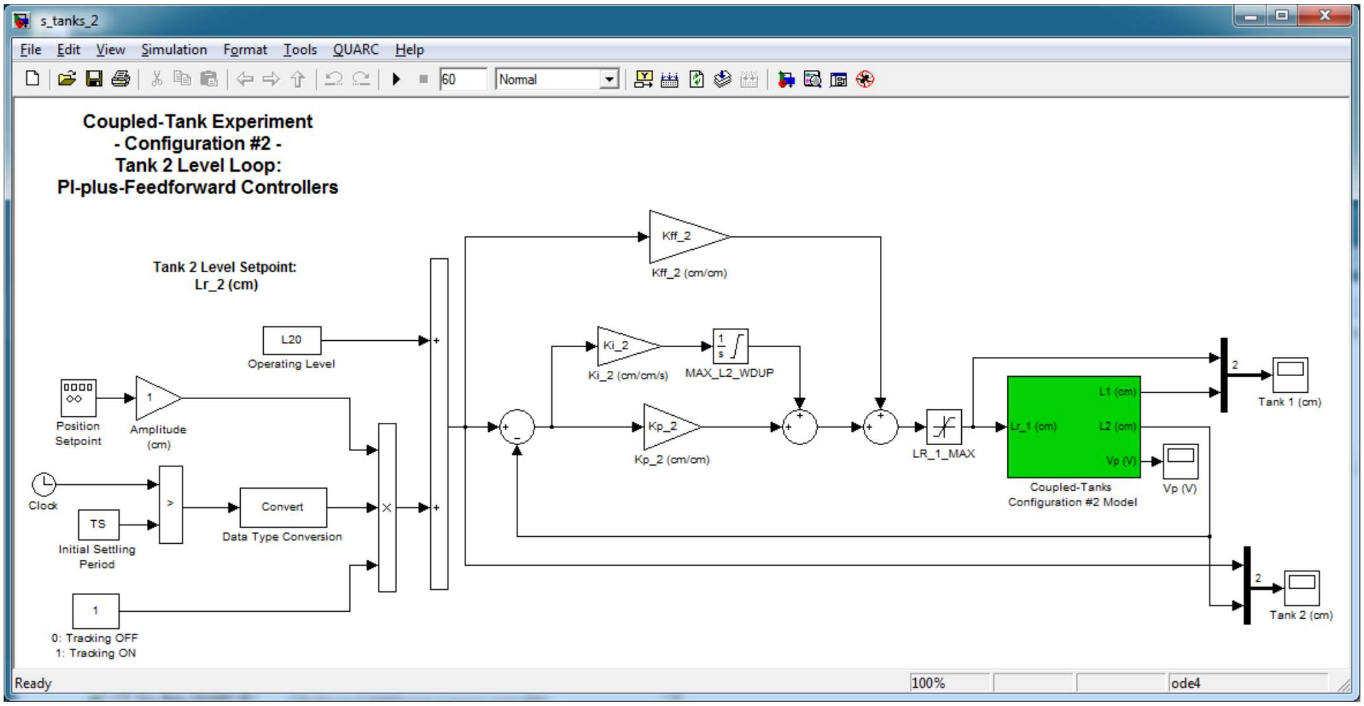


Figure 9.2: Simulink model used to simulate PI-FF control on Coupled Tanks system.

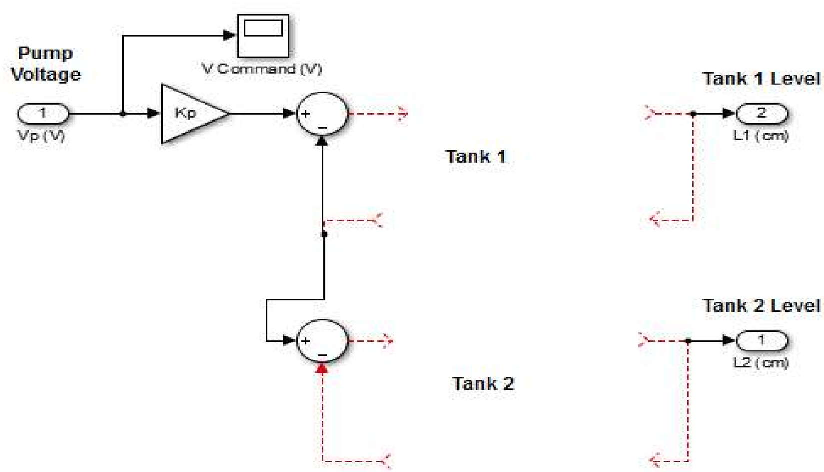


Figure 9.3: Coupled-Tank Non-Linear Model (Subsystem)

Refer to figure 9.3 to create a subsystem in figure 9.2 (Subsystem shown in green)

Follow this procedure:

1. Enter the proportional, integral, and feed-forward gains in MATLAB found in the Tank 1 Control pre-lab questions as *Kp_1*, *Ki_1*, and *Kff_1*.
2. Enter the proportional, integral, and feedforward gain control gains found in Matlab as *Kp_2*, *Ki_2*, and *Kff_2*.
3. To generate a step reference, go to the Signal Generator block and set it to the following:
    - Signal type = *square*

• Amplitude = 1

• Frequency = 0.02 Hz

4. Set the *Amplitude (cm)* gain block to 1 to generate a step that goes between 14 and 15 mm (i.e., ± 1 cm square wave with *L*10 = 15 cm operation point).

   Coupled-Tank Non-Linear Model subsystem has to be created based on its mathematical model (Block shown in green).
5. Open the Tank 1 (cm), Tank 2 (cm), and Vp (V) scopes.
6. Start the simulation. The scopes should be displaying responses similar to Figure 9.4. Note that in the Tank 1 (cm) and Tanks 2 (cm) scopes, the yellow trace is the setpoint (or command) while the purple trace is the simulation.

   Data Saving: Similarly as with s_tanks_1, after each simulation run each scope automatically saves their response to a variable in the Matlab workspace. The Tank 2 (cm) scope saves its response to the data_L2 variable. The Tank 1 (cm) scope saves its response to the variable called data_L1 and the Pump Voltage(V) scope saves its data to the data_Vp variable.
7. Generate a MATLAB figure showing the *Simulated Configuration #2* response. Include both tank 1 and 2 level responses as well as the pump voltage.
8. Measure the steady-state error, the percent overshoot and the settling time of the simulated response. Does the response satisfy the specifications given in Section 2.1.4?

   Hint: Use the Matlab *ginput* command to take measurements off the figure.

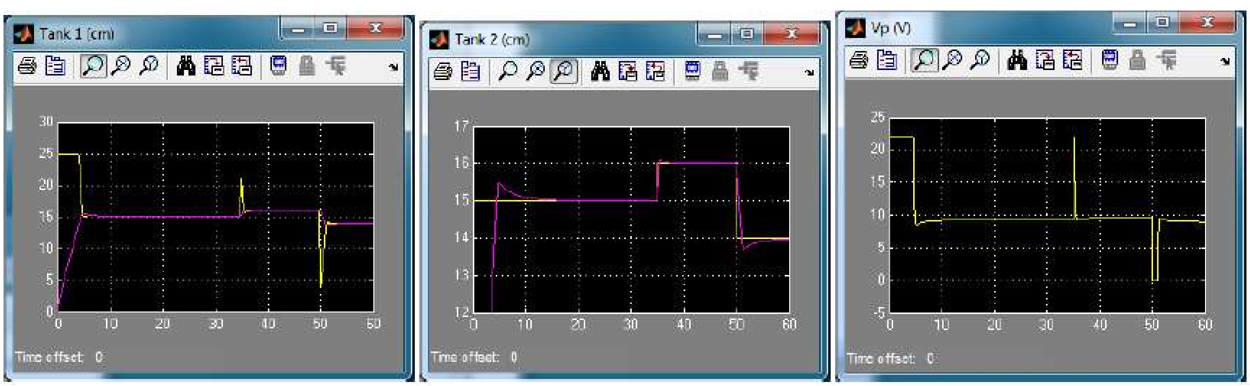


Figure 9.4: Simulated closed-loop tank 2 level control response

VI. Tank 2 Level Control Implementation

The q_tanks_2 Simulink diagram shown in Figure 9.5 is used to run the Tank 2 Level control presented in Section above on the Coupled Tanks system. The *Tank 1 Inner Loop* subsystem contains the PI+FF control used previously in Section 3.3.3 as well as the *Coupled Tanks* subsystem, which contains QUARC blocks that interface with the pump and pressure sensors of the Coupled Tanks system.

Experimental Setup

The *q_tanks_2* Simulink diagram shown in Figure 9.5 will be used to run the feed-forward and PI level control on the actual Coupled Tanks system.

Follow this procedure:

1. Enter the proportional, integral, and feed forward control gains obtained, in Matlab Kp_1, Ki_1 and Kff_1.
2. Enter the proportional, integral, and feed-forward control gains of tank 2 as *Kp_2*, *Ki_2*, and *Kff_2*.
3. To generate a step reference, go to the Signal Generator block and set it to the following:
   - Signal type = square
   - Amplitude = 1
   - Frequency = 0.02 Hz
4. Set the *Amplitude (cm)* gain block to 1 to generate a step that goes between 14 and 15 mm (i.e., $\pm$ 1 cm square wave with $L10$ = 15 cm operation point).
5. Open the *Tank 1 (cm)*, *Tank 2 (cm)*, and *Vp (V)* scopes.

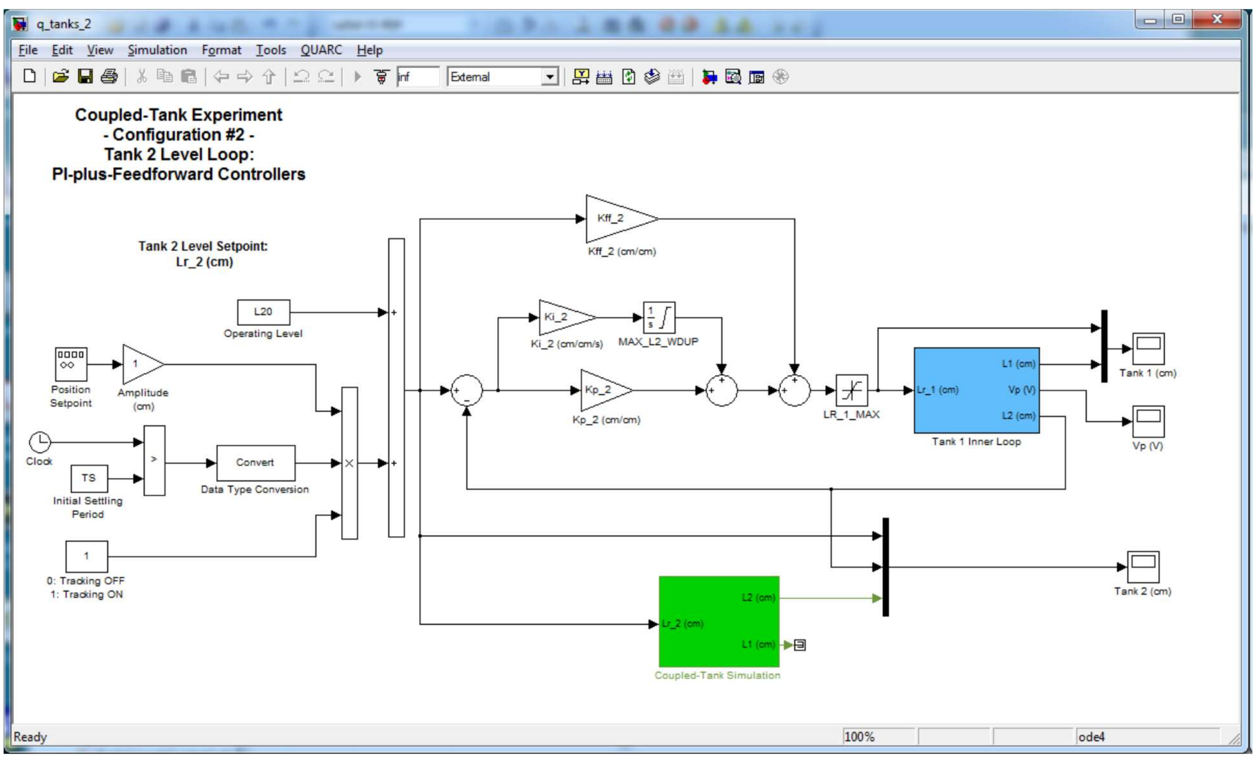
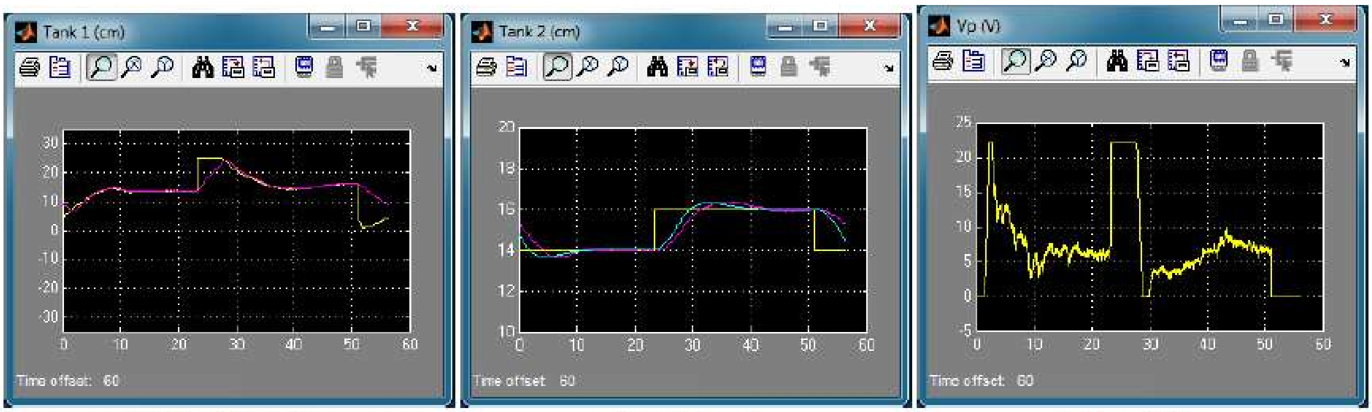


Figure 9.5: Simulink model used to run tank 1 level control on Coupled Tanks system.

6. In the Simulink diagram, go to QUARC | Build.
7. Click on QUARC | Start to run the controller. The level in tank 2 will first stabilize to the operating point tank 2 operating point. After the settling period, the $\pm$ 1 cm step will start. The scopes should be displaying responses similar to Figure 9.6 (after the settling period).

8. Generate a Matlab figure showing the *Implemented Tank 2 Level Control* response, i.e., the tank 1 and 2 levels as well as the pump voltage.

9. Measure the steady-state error, the percent overshoot and the peak time of the response obtained on the actual system



(a) Tank 1 Level (b) Tank 2 Level (c) Pump Voltage

Figure 9.6: Typical response when controlling tank 2 level

VII. Experimental Procedure

1.
   a. Analyze tank 2 water level closed-loop system at the static equilibrium point (L10; L20) and determine and evaluate the voltage feedforward gain, Kff_2.
   b. Using tank 2 voltage-to-level transfer function G2(s) determined in Section 2 and the control scheme block diagram illustrated in Figure 9.1, derive the normalized characteristic equation of the water level closed-loop system.
   c. By identifying the controller gains *Kp*_2 and *Ki*_2, fit the obtained characteristic equation to the standard second-order equation. Determine *Kp*_2 and *Ki*2 as functions of the second order system parameters.
   d. Determine the numerical values for *Kp*_2 and *Ki*_2 in order for the tank 2 system to meet the closed loop desired specifications, as previously stated.
2. Complete the steps listed in Section V of this manual. Use the reference image
3. Complete the steps listed in Section VI of this manual. Use the image reference.